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Credit risk and dynamic capital structure choice

Thomas Dangl [a] and Josef Zechner [b,*]

[a] *Department of Managerial Economics and Industrial Organization, Vienna University of Technology, Theresianumgasse 27, A-1040 Vienna, Austria*
[b] *Department of Business Studies, University of Vienna, Brünner Straße 72, A-1210 Vienna, Austria*

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Abstract

This paper analyzes the effect of dynamic capital structure adjustments on credit risk. Firms may optimally adjust their leverage in response to stochastic changes in firm value. It is shown that capital structure dynamics lower optimal initial leverage ratios but increase both, fair credit spreads and expected default probabilities for moderate levels of transactions costs. Numerical examples demonstrate that expected default frequencies do not decrease monotonically in the traditional distance to default measure. The magnitude of the effect of capital structure dynamics depends on firm characteristics such as asset volatility, the growth rate, the effective corporate tax rate, debt call features and transactions costs. We find that the underestimation of credit spreads and expected default frequencies is exacerbated when the risk-adjusted drift of the underlying stochastic process is inferred from a model which ignores the opportunity to recapitalize. Finally it is shown that the value-at-risk of corporate bonds increases with the distance to default (DD) both for very low and for very high values of DD whereas it decreases for intermediate values.

JEL classification: G3; G2; G32; G28; G38

Keywords: Risk management; Credit risk; Capital structure

1. Introduction

Measuring and managing credit risk has become of central importance for financial institutions. In most countries, banks' equity requirements are already tied to their exposure to credit risk. According to the proposed Basel Accord II, the link between credit risk

* Corresponding author.
E-mail addresses: thomas.dangl@tuwien.ac.at (T. Dangl), josef.zechner@univie.ac.at (J. Zechner).

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and capital requirement will be regulated in much more detail. Banks will be allowed to calculate their credit risk exposure and thus their equity requirements on the basis of their internal rating models.

Perhaps even more importantly, the search for shareholder value requires that banks can accurately quantify their exposures to unexpected credit losses. This is a prerequisite for a correct allocation of economic capital to various lending activities and thus for optimizing capital budgeting decisions.

Despite their importance for regulation and the management of financial institutions, existing credit risk models are still unable to capture some important risk factors. For example, most existing credit risk models assume that the firm's debt level remains constant over time or changes in a deterministic way. In practice firms adjust their financial structures in response to stochastic changes in their economic environment. This may have significant influence on credit risk, so the question is: how do we account for this in a credit risk model?

In this paper we show how firms' dynamic capital structure choices can be integrated into a credit risk model. We analyze the effect of intertemporal capital structure choices on a corporate bond's fair credit spread, on estimated distances to default, on expected default frequencies and on the bond's value-at-risk.

We present a model where the firm's free cash flow follows a geometric Brownian motion. This cash flow is partly used to pay the coupon on the firm's debt and the remainder is paid out as a dividend to equityholders.

Debt is advantageous for tax reasons. The net tax advantage of debt is the difference between the corporate tax advantage of debt (interest is corporate tax deductible) and the personal tax disadvantage of debt (interest income is taxed more heavily than capital gains or dividends).[1]

Recapitalizations are associated with transactions costs. As a result firms do not adjust their capital structures continually. If the free cash flow increases by a sufficient amount, then the firm may find it optimal to issue more debt. Since the risk-free rate of interest is assumed constant and since the new optimal leverage ratio is equal to the initially-chosen leverage ratio the new debt can be issued at precisely the same terms as the original debt.

We contrast our model of dynamic recapitalization with the traditional approach in the spirit of Merton (1974) where the face value of debt at the risk horizon is assumed to be fixed. For low or moderate level of transactions costs we find that consideration of dynamic recapitalization decisions generally increases fair credit spreads and the expected default frequencies. Interestingly, we find a nonmonotonic, U-shaped relationship between distance to default and expected default frequencies. One of the major implications of our analysis is that it would be wrong to estimate an unconditional empirical relationship between distance to default and expected default frequencies. Our results indicate that one must condition on the firm's asset volatility, its effective corporate tax rate, its expected growth rate and estimated bankruptcy costs.

[1] Interest is taxable at the personal level whereas the realized rate of return on equity is not. This is so since we assume that the rate of return on equity is either realized in the form of tax free capital gains or realized in the form of dividends which are not taxed because of imputation of the corporate tax rate.

The analysis also reveals that the riskiness of a corporate bond does not increase monotonically as the issuing firm's distance to default decreases. By contrast, in a model with dynamic recapitalizations the value-at-risk of a corporate bond increases with the distance to default when the firm is very close to bankruptcy and also when the firm is very far away from bankruptcy.

Our analysis is related to several papers. As in Fischer et al. (1989a) we explicitly model the possibility of dynamic capital structure changes. We extend the analysis to focus on the impact of dynamic capital structure adjustments on fair credit spreads, expected default frequencies, and a bond's value-at-risk. Also, we use the firm's cash flow as the state variable, rather than the value of the firm's unlevered assets, as in Fischer et al. (1989a).

Goldstein et al. (2001) extend Fischer et al. (1989a) by modeling the firm value contingent on its cash flow rather than on the value of its unlevered assets. While our model is also a cash flow based model of the firm, we explicitly allow firms to be optimally relevered after default. Most importantly, however, we extend the analysis of Fischer et al. (1989a) and Goldstein et al. (2001) by exploring the effects of capital structure dynamics on risk measures which are widely used in credit risk models and in corporate bond portfolio management. Specifically, we focus on the effects of dynamic capital structure choice on expected default frequencies, a corporate bond portfolio's value-at-risk, and on the relationship between a firm's distance to default and fair credit spreads.

Christensen et al. (2000) develop a model of dynamic capital structure adjustments. They explicitly explore the impact of renegotiations between equityholders and debtholders in times of financial distress whereas we do not allow for such renegotiations.

Collin-Dufresne and Goldstein (2001) analyze whether or not credit spreads reflect stationary leverage ratios. In their model, leverage ratios are mean reverting. Consistent with empirical evidence they find that in comparison to a model with constant leverage, debt issued by low-leverage firms has larger credit spreads and that the term structure of debt is upward sloping for low-grade debt.

We extend the analysis of the effects of dynamic leverage adjustments on credit spreads in Collin-Dufresne and Goldstein (2001) by explicitly modeling equityholders' optimal capital structure choices. This allows us to explore how the effect of capital structure dynamics on credit spreads is related to the characteristics of issuing firms.

The remainder of the paper is organized as follows. Section 2 introduces the model. The results of the analysis are presented in Section 3. Section 4 summarizes and concludes.

2. The model

We assume that the firm's instantaneous free cash flow after corporate tax, c_t, follows a geometric Brownian motion given by

$$\frac{\mathrm{d}c_t}{c_t} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \qquad c_0 = c(0), \tag{1}$$

where the expected drift rate and the instantaneous variance of the cash flow process are defined by μ and $c_t^2\sigma^2$, respectively (see Table 1 for the notation used throughout the paper), and $\mathrm{d}W_t$ is the increment to a standard Wiener process. Hence, if r and $\hat{\mu}$ denote the

Table 1
Notation

A firm's instantaneous free cash flow after corporate tax	c_t
Expected rate of change of c_t	μ
Risk-adjusted drift of the cash flow process	$\hat{\mu}$
Riskless rate of interest	r
Instantaneous variance of the cash flow process	$c_t^2\sigma^2$
Face value of debt	B
Value of equity	E
Value of debt	D
Total value of the firm	V
Instantaneous coupon rate	i
Firm's inverse leverage ratio	y_t
Personal tax rate on ordinary income	τ_p
Corporate tax rate	τ_c
Proportional bankruptcy costs	g
Proportional transactions costs associated with issuing new debt	k
Proportional call premium	λ

riskless interest rate and the risk-adjusted drift rate of the cash flow process,[2] respectively, and if equity income is not taxed but interest income is taxed at the personal tax rate τ_p then the current value of the unlevered cash flow is given by $c_t/(r(1-\tau_p)-\hat{\mu})$.

We assume that the effective corporate tax rate τ_c exceeds τ_p. Thus, given that coupon payments are tax deductible, firms have an incentive to issue debt and to adjust their optimal capital structure over time.[3] Let B_t denote the face value of outstanding debt at time t, which is endogenously determined by the decisionmakers within the firm. We define

$$y_t = \frac{1}{B_t}\frac{c_t}{r(1-\tau_p)-\hat{\mu}} \tag{2}$$

as the inverse leverage ratio with respect to the value of the firm's unlevered assets.

Since we assume that it is costly to call outstanding debt (call premium λ) as well as to issue new debt (proportional transactions costs k), it is not optimal to adjust the capital structure continuously. Rather than reacting to any change in the firm's leverage ratio, only sufficient deviations from the optimum satisfy the expenses associated with a reorganization of debt (see Fischer et al. (1989a) and Fischer et al. (1989b)). Consequently, the risk-neutral dynamics of the inverse leverage ratio y_t is given by

$$\begin{aligned} \frac{\mathrm{d}y_t}{y_t} &= \begin{cases} \hat{\mu}\,\mathrm{d}t + \sigma\,\mathrm{d}W_t: & \text{no debt reorganization at time } t, \\ B_t/B_t^* - 1: & \text{debt is restructured from } B_t \text{ to } B_t^* \text{ at time } t, \end{cases} \\ y_0 &= y(c_0, B_0) = \frac{1}{B_0}\frac{c_0}{r(1-\tau_p)-\hat{\mu}}, \end{aligned} \tag{3}$$

[2] Assuming that the cash flow process is observable, the risk-adjusted drift can be inferred from the value of a traded security contingent on the firms cash flow, such as equity. In this estimation procedure the model assumptions (e.g., about the reorganization of corporate debt) play a crucial role; see Section 3.2 for a discussion of issues related to model risk.

[3] Kemsley and Nissim (2002) empirically estimate the value of the debt tax shield with approximately 40% of debt balance, or 10% of firm value.

that is, during periods where the amount of debt issued is constant, the inverse leverage ratio follows the same geometric Brownian dynamics as the cash flow process c_t. Whenever the firm's management finds it optimal to reorganize debt, the face value B_t jumps to the amount of newly issued debt and analogously there is a jump in the inverse leverage ratio y_t.

In the remainder we interpret the firm's equity E and debt D as claims contingent on y_t and B_t rather than as claims contingent on the firm's profit flow c_t. This construction allows us to formulate the entire model to be homogeneous in B, i.e., $E(y, B) = BE(y, 1)$ and $D(y, B) = BD(y, 1)$. The reason is the fact that the cash flow $c_t = (r(1 - \tau_p) - \hat{\mu})y_t B_t$ as well as payments in the case of debt restructuring are proportional to B_t. The assumption of proportional bankruptcy costs preserves this homogeneity. Thus, the endogenously-chosen coupon rate i is constant, i.e., the coupon payment is also proportional to B_t.

We consider reorganization strategies determined by an upper threshold $\bar{y}$ and a lower threshold $\underline{y}$ for the inverse leverage ratio [4]. This means whenever y_t reaches $\bar{y}$, the amount of outstanding debt is increased by calling existing debt and issuing new debt with higher face value. Whenever y_t reaches $\underline{y}$, equityholders decide to default. In the absence of debt renegotiations, equityholders will never find it optimal to recapitalize in response to decreases in firm value. The intuition is that risky debt can be interpreted as a riskless claim plus a short position in a put option that gives equityholders the right to redeem the debt by turning over the firm's assets to the bondholders. When buying back debt in bad states, firms would give up this option and reduce interest tax shields.[5] Equityholders do therefore not find it optimal to reduce leverage after the firm's value has deteriorated. This is independent of the magnitude of bankruptcy costs since they are borne by the bondholders ex post and thus do not influence equityholders' decision.[6] Allowing renegotiations with the bondholders could make debt reduction in the form of a voluntary debt forgiveness by bondholders optimal. While debt renegotiation is certainly an interesting feature of dynamic capital structure policies, we only consider capital structure changes where bondholders get their full claims when capital structure is adjusted outside of formal bankruptcy.[7] There is empirical support for this feature of our model. For example, Gilson (1997) finds that debt reduction outside Chapter 11 is rarely observed. Due to homogeneity it is always optimal to establish the same optimum leverage ratio, denoted by $\tilde{y}^*$, in the case of a capital structure adjustment. This means the current amount of debt B_t is kept constant as long as y_t is in the range between $\underline{y}$ and $\bar{y}$. Only if y_t hits $\bar{y}$, B jumps to $(\bar{y}/\tilde{y}^*)B_t$. If equityholders default, the ownership is transferred to the bondholders. After paying bankruptcy costs they optimally relever the firm, i.e., y immediately jumps to $\tilde{y}^*$. Of course, we require $\underline{y} < \tilde{y}^* < \bar{y}$. Figure 1 plots one particular realization of y_t which

[4] Harrison et al. (1988) show the optimality of these control-limits strategies for controlling Brownian motions in the presence of lump-sum costs.

[5] More precisely, by reducing leverage equityholders would reset the exercise price of the put option they own at a lower level, i.e. at the new debt level after the leverage reduction.

[6] A rigorous proof of this argument is available from the authors upon request.

[7] For a detailed of the impact of debt renegotiation on the value of different claims on the firm's cash flow, see for example, Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Christensen et al. (2000), and Flor (2002).

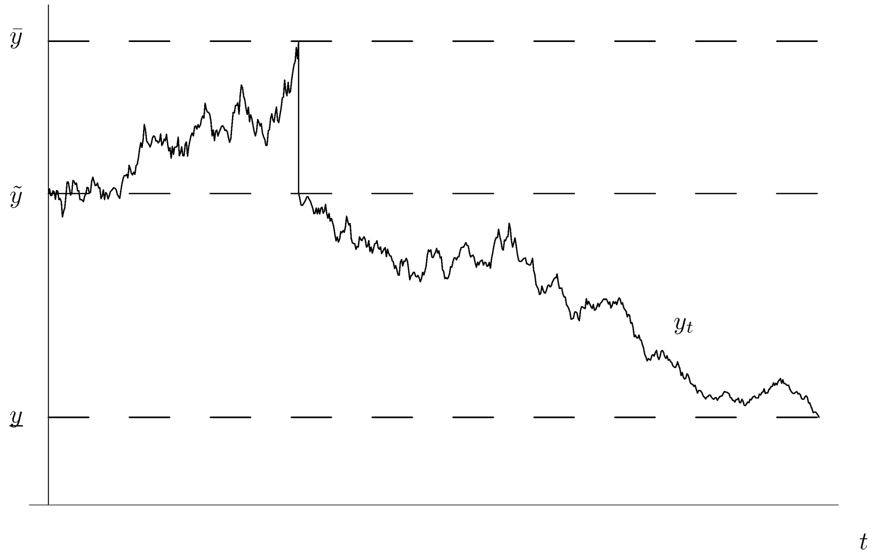

Fig. 1. One particular realization of y_t. When the firm reorganizes its debt (when y_t hits $\bar{y}$), the inverse leverage ratio jumps to $\tilde{y}$. This jump reduces the distance of y_t to the critical default threshold $\underline{y}$, and thus, increases the default probability of the firm that dynamically adjusts its capital structure.

illustrates the characteristics of the dynamics of the firm's inverse leverage ratio. At a recapitalization (when y_t hits $\bar{y}$) y_t jumps to $\tilde{y}$, thereby reducing the distance to the critical default trigger $\underline{y}$.

2.1. The value of equity and debt

In this section we consider a given (not necessarily optimal) reorganization strategy $(\underline{y}, \tilde{y}, \bar{y})$ and determine the value of equity and debt. Based on these results we subsequently determine the optimal strategy in Section 2.2.

Since B is kept constant in the interval $(\underline{y}, \bar{y})$, we can apply standard contingent claims valuation techniques to determine the value of equity $E(y, B)$ and debt $D(y, B)$. More precisely, when the face value of debt issued is constant, $B_t = B$, the value of equity and debt must satisfy

$$\frac{1}{2}\sigma^2 y^2 E_{yy} + \hat{\mu} y E_y - r(1-\tau_p)E - (1-\tau_c)iB + \big(r(1-\tau_p) - \hat{\mu}\big)y_t B = 0, \tag{4}$$

$$\frac{1}{2}\sigma^2 y^2 D_{yy} + \hat{\mu} y D_y - r(1-\tau_p)D + (1-\tau_p)iB = 0. \tag{5}$$

The solutions to these second-order ordinary differential equations are

$$E(y, B) = BE_1 y^{m_1} + BE_2 y^{m_2} - \frac{(1-\tau_c)i}{(1-\tau_p)r}B + yB, \tag{6}$$

$$D(y, B) = BD_1 y^{m_1} + BD_2 y^{m_2} + \frac{i}{r}B, \tag{7}$$

where m_1 and m_2 are the positive and the negative root of the characteristic quadratic polynomial, i.e.,

$$m_{1,2} = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2}\right)^2 + \frac{2r(1-\tau_p)}{\sigma^2}}, \tag{8}$$

and $E_{1,2}$ and $D_{1,2}$ are constants that are determined by the conditions

$$E(\underline{y}, B) = 0, \tag{9}$$

$$E(\bar{y}, B) = \left[V\left(\tilde{y}, B\frac{\bar{y}}{\tilde{y}}\right) - kB\frac{\bar{y}}{\tilde{y}}\right] - (1+\lambda)B, \tag{10}$$

where V denotes the total value of the firm, $V = E + D$. Equation (9) states that equity is worthless in the case of default.[8] When the firm is recapitalized (Eq. (10)), it first buys back the outstanding debt securities, paying $(1 + \lambda)B$. After that the firm immediately relevers optimally, i.e., it issues new debt with a face value $(\bar{y}/\tilde{y})B$. This is associated with transactions costs equal to $kB\bar{y}/\tilde{y}$ and is reflected by the first term of Eq. (10).

The boundary conditions for debt valuation are

$$D(\underline{y}, B) = \left[V\left(\tilde{y}, B\frac{\underline{y}}{\tilde{y}}\right) - kB\frac{\underline{y}}{\tilde{y}}\right](1-g), \tag{11}$$

$$D(\bar{y}, B) = (1+\lambda)B. \tag{12}$$

On default (Eq. (11)), the bondholders become owners of the firm which they immediately relever optimally. The proportional bankruptcy costs are borne by the new owners of the firm. When the firm is recapitalized (Eq. (12)), the outstanding debt is called back at the price $(1 + \lambda)B$.

Since we assume that debt is always issued at par, we determine the coupon rate i endogenously:

$$\text{choose } i \text{ such that } D(\tilde{y}) = B. \tag{13}$$

2.2. Optimal recapitalization

In the previous subsection we have derived the value of equity and debt under a given recapitalization strategy $(\underline{y}, \tilde{y}, \bar{y})$. Now we wish to determine the optimal choice of these critical values. We note that the optimal reorganization point $\bar{y}^*$ and the default point $\underline{y}^*$ will be chosen to maximize the value of equity.[9] By contrast, the optimal leverage ratio $\tilde{y}^*$ will be chosen to maximize the total value of the firm. This is so since $\tilde{y}^*$ is chosen when the firm is unlevered and, thus, it is in the firm owners' best interest to choose $\tilde{y}^*$ to maximize total firm value. Precisely, for a given $\tilde{y}$, equityholders will optimize their decision

[8] Our model can be extended to allow for renegotiation between equityholders and bondholders to avoid bankruptcy costs. Such renegotiations have been modeled, e.g., by Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Christensen et al. (2000), and Flor (2002).

[9] See Section 2 for a discussion why it is—in the absence of renegotiation—not optimal for equityholders to repurchase outstanding debt in response to a decrease in firm value.

variables $\underline{y} = \underline{y}^*(\tilde{y})$ and $\overline{y} = \overline{y}^*(\tilde{y})$ which simultaneously satisfy the first-order conditions of optimality (see Dixit (1993) for a discussion of the so called 'smooth pasting' conditions):

$$\frac{\partial E}{\partial y}(\underline{y}, B) = 0, \tag{14}$$

$$\frac{\partial E}{\partial y}(\overline{y}, B) = \frac{1}{\tilde{y}}\left[E(\tilde{y}, B) + B(1-k) + \overline{y}\frac{\mathrm{d}E}{\mathrm{d}\overline{y}}(\tilde{y}, B)\right]. \tag{15}$$

When issuing new debt, thereby fixing the coupon rate i to issue the bond at par, the owner of the unlevered firm anticipates the recapitalization strategy and chooses the optimal initial capital structure by solving

$$\max_{\tilde{y}} V(\tilde{y}, B) - kB, \tag{16}$$

subject to

$$B = \frac{1}{\tilde{y}}\frac{c_0}{r(1-\tau_p) - \hat{\mu}}, \qquad \underline{y} = \underline{y}^*(\tilde{y}), \qquad \overline{y} = \overline{y}^*(\tilde{y}), \qquad i\colon D(\tilde{y}, B) = B.$$

Therefore, the first-order condition that has to be satisfied by the optimal initial inverse leverage ratio $\tilde{y}^*$ is

$$\frac{\partial V}{\partial y}\bigg|_{(\tilde{y}^*, B)} + \frac{\partial V}{\partial \tilde{y}}\bigg|_{(\tilde{y}^*, B)} + \frac{\partial V}{\partial \overline{y}^*}\frac{\partial \overline{y}^*}{\partial \tilde{y}}\bigg|_{(\tilde{y}^*, B)} + \frac{\partial V}{\partial \underline{y}^*}\frac{\partial \underline{y}^*}{\partial \tilde{y}}\bigg|_{(\tilde{y}^*, B)} + \frac{\partial V}{\partial i}\frac{\partial i}{\partial \tilde{y}}\bigg|_{(\tilde{y}^*, B)} - \frac{1}{\tilde{y}^*}\big(V(\tilde{y}^*, B) - kB\big) = 0. \tag{17}$$

2.3. A benchmark: the case of a constant debt level

As a benchmark we use the case where the firm is not allowed to reorganize debt, i.e. $B_t = B_0$, the initially-chosen amount of debt cannot be changed. As a consequence, the set of decision variables contains only the initial capital structure $\tilde{y}$ and the lower critical value $\underline{y}$. The absence of the reorganization opportunity is further reflected in a change in the boundary conditions. Since there exists no upper threshold that triggers a jump in the capital structure, condition (10) has to be substituted by

$$\lim_{y\to\infty} E(y, B) = -\frac{(1-\tau_c)i}{(1-\tau_p)r}B + y_t B, \tag{18}$$

and condition (12) by

$$\lim_{y\to\infty} D(y, B) = \frac{i}{r}B. \tag{19}$$

Equation (18) states that if the inverse leverage ratio goes to infinity, then the equity value equals the total value of the firm minus the present value of all future after-tax payments to bondholders. For infinitely large values of y, the value of debt becomes the present value of all future coupon payments to bondholders. Note that in Eq. (19) the term $1 - \tau_p$ drops out since it appears in both the numerator and the denominator.

Since E and V are now independent of $\bar{y}$ the optimality condition (15) has to be dropped, and $\partial V/\partial\bar{y} = 0$ can be substituted into condition (17).

3. Results and comparative statics

This section presents the results of the model analysis. It consists of three subsections, each starting from a common base case scenario (see Table 2 for the parameter values) and deriving different comparative statics results. The first (Section 3.1) focuses on the firm's optimal capital structure choice when it is allowed to dynamically reorganize debt compared to the benchmark model with a static debt level. Since we endogenously determine the firm's capital structure choice, we are able to explore the impact of firm characteristics (like the variance of the cash flow process, the growth rate, or the tax advantage of debt) on fair credit spreads and the optimal initial leverage ratio. The second (Section 3.2) concentrates on model risk from an analyst's point of view. Specifically, we estimate credit risk from observed equity time series and examine the impact of the model choice on the calculated credit spreads. In the third (Section 3.3) we determine expected default frequencies implied by a dynamic capital structure choice. Furthermore, we explore the impact of firm characteristics on the relationship between distance to default and expected default frequencies.

3.1. The firm's optimal capital structure choice

Table 3 shows the optimal recapitalization strategy of a firm with dynamic capital structure adjustments together with the optimal choice of a firm with static debt level. While $1/y$ is the leverage ratio of the firm with respect to the value of the unlevered firm, $B/V(y)$ defines the leverage ratio as the face value of debt divided by the total value of a firm that follows an optimal dynamic capital structure policy. Thus, $B/V(y)$ accounts for the total market value of the levered firm, correctly reflecting the present value of tax shields, transactions costs due to capital structure adjustments and bankruptcy costs. The most evident difference is that a dynamic capital structure strategy initially uses much less debt than a static strategy does. The dynamic recapitalization strategy anticipates the fact that debt will be increased if the firm value increases by a sufficient amount. It finds the optimal choice

Table 2
Base case parameters

Parameter	Value, %
Riskless rate of interest, r	5
Personal tax rate, τ_p	35
Corporate tax rate, τ_c	50
Variance, σ_y^2	5
Risk-adjusted drift, $\hat{\mu}$	0
Transactions costs, k	1
Call premium, λ	0
Bankruptcy costs, g	25

Table 3
Optimal capital structure choice (base case)

Parameter	Dynamic, %	Static, %
Optimal initial leverage ratio[a], $1/\tilde{y}^*$	58.6	70.0
Initial leverage ratio[b], $B/V(\tilde{y}^*)$	50.9	63.3
Max. leverage ratio[a], $1/\underline{y}^*$	207.9	204.6
Max. leverage ratio[b], $B/V(\underline{y}^*)$	242.4	248.5
Min. leverage ratio[a], $1/\bar{y}^*$	39.3	0.00
Min. leverage ratio[b], $B/V(\bar{y}^*)$	34.4	0.00
Coupon rate, $i(\tilde{y}^*)$	7.75	7.44

[a] w.r.t. the unlevered assets.
[b] w.r.t. the optimally levered firm.

Table 4
Comparative static analysis

	$1/\tilde{y}^*$ (dyn., %)	$B/V(\tilde{y}^*)$ (dyn., %)	i (dyn., %)	$1/\tilde{y}^*$ (stat., %)	$B/V(\tilde{y}^*)$ (stat., %)	i (stat., %)	$\Delta(B/V)$ (%)	Δi (bp)
$\sigma_y^2 = 0.05$	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
0.04	60.6	52.7	7.30	71.8	64.6	7.06	−11.9	24
0.02	67.9	58.7	6.35	77.9	68.8	6.23	−10.1	12
$\tau_c - \tau_p = 0.15$	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
0.11	45.2	42.0	7.03	56.7	53.6	6.92	−11.6	11
0.05	22.4	22.0	5.93	29.9	29.5	5.97	−7.5	−4
$k = 1$	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
4	55.5	49.6	7.26	64.7	58.7	7.17	−9.1	9
8	49.2	44.9	6.76	56.3	51.5	6.80	−6.6	−4
$g = 5$	107.0	80.3	11.12	102.3	86.8	8.72	−6.5	240
15	73.3	61.2	8.45	82.5	73.0	7.86	−11.8	59
25	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
$\hat{\mu} = -2$	54.7	49.5	8.56	66.7	60.9	8.39	−11.4	17
0	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
2	74.0	53.3	7.04	74.2	66.3	6.67	−13.0	37
$\lambda = 0$	58.6	50.9	7.75	70.0	63.3	7.44	−12.4	31
5	61.5	53.5	7.40	70.0	63.3	7.44	−9.8	−4
10	63.8	55.5	7.28	70.0	63.3	7.44	−7.8	−16

by balancing the tax benefits of debt against the costs of debt including the costs associated with recapitalization. By contrast, when a firm with static capital structure policy wants to take full advantage of the tax benefits, it initially has to take a larger amount of debt to account for the possible favorable future evolution of its asset value. Counterintuitively, the fair coupon rate under the dynamic capital structure strategy exceeds that under the static one. The reason is the fact that under dynamic capital structure the firm issues more debt when the firm value increases. This eliminates the chance for bondholders that the value of their contract can rise significantly above par.

Table 4 lists comparative statics on σ^2, $\tau_c - \tau_p$, k, g, $\hat{\mu}$, and λ. As discussed above, we find that the opportunity to recapitalize reduces the optimal initial leverage ratio $B/V(\tilde{y}^*)$.

Despite this effect, dynamic recapitalization increases credit spreads as long as transactions costs associated with debt repurchase are low (i.e., if k and/or λ is low compared to $\tau_c - \tau_p$).[10] The excess in credit spreads and the reduction in the leverage ratio is more pronounced for high-risk firms and for firms with high growth rates. If the costs for capital structure adjustments increase, then the dynamic firm's credit spreads are lower than for the static capital structure policy.

If the possibility to renegotiate would be introduced, then bondholders and equityholders could bargain after a deterioration of firm value to avoid deadweight bankruptcy costs. This effectively raises the net benefit of debt and would lead to higher optimal initial leverage ratios and to lower bounds for leverage-increasing capital structure changes, i.e., a lower $\bar{y}$. Thus, allowing leverage-reducing renegotiations on the firm's capital structure choice would effectively eliminate bankruptcy costs. The direction of the influence of debt renegotiation in our model can therefore be seen from Table 4 when we vary the size of bankruptcy costs. We see that a reduction of bankruptcy costs from 25% to 5% increases the optimal initial leverage ratio, $B/V(y)$, from 50.9% to 80.3%. At the same time the coupon rate rises from 7.75% to 11.12%. Furthermore, we find that for bankruptcy costs equal to 25%, the firm issues additional debt when the leverage ratio $B/V(y)$ reaches 34.4% whereas the firm already recapitalizes at 56.9% when bankruptcy costs are only 5%.

Inspection of the critical leverage ratio that leads to default reveals that a firm with bankruptcy costs equal to 25% defaults at a leverage ratio of 242.4%. By contrast, this critical ratio is only 145.5% when the bankruptcy cost is 5%. Thus, firms with lower bankruptcy costs exhibit a significantly higher frequency of recapitalizations.

One can also see from Table 4 that the effect of capital structure dynamics on credit spreads is extremely sensitive to the size of bankruptcy costs. For bankruptcy costs of 25%, the credit spread of a bond issued by a firm that cannot adjust leverage is only 31 basis points lower than the credit spread of a bond issued by a firm that adjusts leverage dynamically. This difference increases to 240 basis points when bankruptcy costs are only 5%!

Summarizing, we can conclude that introducing the possibility of renegotiating debt after the firm value has decreased would have effects which are similar to the effect of reducing bankruptcy costs. First, renegotiation would lead to higher optimal initial leverage ratios, higher leverage ratio bounds at which leverage is increased and lower leverage ratio bounds at which debt is adjusted downwards. In general, one can conclude that the effect of capital structure dynamics would be even more significant with debt renegotiation.

3.2. Empirical implementation and model risk

In the previous section we have explored the effect of dynamic recapitalization on leverage choice and credit spreads. We have thereby assumed that the risk-adjusted drift of the

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[10] If the relative costs for debt repurchases increase, a lower coupon helps to reduce the frequency of debt restructuring. However, due to the possibility of future adjustments, the firm which pursues a dynamic capital structure policy can utilize the tax advantage of debt more effectively even if coupons are lower, as it is indicated by higher values of $V(\tilde{y}^*)/(\tilde{y}^* B)$.

cash flow process, $\hat{\mu}$, is directly observable. For practical applications, the risk-adjusted drift must be inferred from the observable market value of equity, E. However, the model of the firm that is used to infer $\hat{\mu}$ may have a crucial effect on the estimate that is obtained. Thus, using a "wrong" model has two effects. First, there is a direct effect since for any given set of model parameters the model generates wrong results. Second, there is also an indirect effect since the model is first used to infer the input parameter $\hat{\mu}$. Thus, the final estimates of fair credit spreads etc. generated by a static model are influenced by both effects. We refer to the combination of these two effects as model risk.

We therefore analyze a credit risk manager who observes the firm's current cash flow, c_t, its volatility,[11] the market value of equity E, and the face value of debt, B and then infers $\hat{\mu}$ and calculates the fair coupon rate of the corporate bond.

Specifically, we proceed as follows to generate the comparative static results presented in Table 5. We assume that the credit risk manager observes c/B, σ^2 and E/B. In the first case (columns 5–7 in Table 5), she uses the dynamic model from Section 2.2 to back out $\hat{\mu}$ and then calculates the implied y/B. Finally, she calculates the fair interest rate, i, again using the dynamic model. In the second case, (columns 8–10) in Table 5) she uses the static model in Section 2.3 to back out $\hat{\mu}$. Then she calculates the implied y/B and solves for the fair interest rate, again using the static model.

Table 5
Comparative static analysis (model risk)

Observations			Dynamic capital structure			Static capital structure				
c/B	σ^2	E/B	$\hat{\mu}$	y/B	i (%)	$\hat{\mu}$	y/B	i (%)	Δi (bp)	$\Delta\hat{\mu}$
Base case										
0.03250	0.05	0.2790	0.00	1.00	10.18	0.00169	1.055	8.75	−143	0.00169
0.04875	0.05	0.7468	0.00	1.50	8.11	0.00246	1.623	6.95	−116	0.00246
0.06500	0.05	1.2825	0.00	2.00	7.41	0.00312	2.212	6.32	−109	0.00312
Low volatility										
0.03250	0.04	0.2698	0.00	1.00	9.38	0.00151	1.049	8.18	−120	0.00151
0.04875	0.04	0.7403	0.00	1.50	7.54	0.00230	1.614	6.56	−97	0.00230
0.06500	0.04	1.2814	0.00	2.00	6.94	0.00299	2.203	6.02	−92	0.00299
Low volatility										
0.03250	0.02	0.2465	0.00	1.00	7.60	0.00106	1.034	6.91	−69	0.00106
0.04875	0.02	0.7321	0.00	1.50	6.32	0.00193	1.595	5.75	−57	0.00192
0.06500	0.02	1.2993	0.00	2.00	5.96	0.00284	2.192	5.40	−56	0.00284
Low growth										
0.05250	0.05	0.2650	−0.02	1.00	11.66	−0.00189	1.023	10.54	−112	0.01811
0.07875	0.05	0.6965	−0.02	1.50	9.19	−0.00178	1.565	8.10	−109	0.01822
0.10500	0.05	1.1988	−0.02	2.00	8.34	−0.00167	2.132	7.19	−115	0.00326
High growth										
0.01250	0.05	0.4621	0.02	1.00	7.96	0.0228	1.294	6.68	−128	0.00284
0.01875	0.05	1.0955	0.02	1.50	6.79	0.0231	1.996	5.84	−95	0.00310
0.02500	0.05	1.7863	0.02	2.00	6.38	0.0233	2.718	5.55	−83	0.00330

[11] The volatility can be estimated from historical observations of c_t.

The results of our numerical examples in Table 5 are unambiguous. Using the static capital structure model instead of a model which accounts for possible debt adjustments leads to a systematic overestimation of the risk-adjusted drift $\hat{\mu}$ (or in other words, to an underestimation of the market price for risk). This overestimation of $\hat{\mu}$ then results in a systematic overestimation of the current value of the unlevered cash flow, and thus, in an overestimation of the inverse leverage ratio y. It directly follows that this estimation error causes an underestimation of fair credit spreads. This underestimation is higher when credit risk is high (i.e., when the level of the cash flow process is low). Thus, the empirical estimation of $\hat{\mu}$ exacerbates the misprizing implied by models which do not account for capital structure dynamics.

The underestimation of required credit spreads also depends on the volatility of the cash flow. While the error in the estimation of the risk-adjusted drift does not significantly depend on the volatility of the cash flow, we see that for firms with high cash flow volatility the underestimation of fair credit spreads is more pronounced.

Finally, for low-growth firms the underestimation of fair credit spreads due to the application of a model that supposes a static level of corporate debt is more severe than for high-growth firms.

3.3. Theoretical expected default frequency

In this section we examine the impact of a dynamic capital structure strategy on the default probability. We calculate the theoretical expected default frequency (TEDFs) of a firm, which we define as the probability that the firm defaults within a certain time period of length T. When focusing on the pricing of credit risk, we are interested in the risk-neutral TEDF which is computed with respect to the risk-neutral probability measure since this is relevant for pricing debt issues (see e.g. Alexander, 1998).[12]

If no recapitalization is allowed, the probability that the firm with initial inverse leverage ratio y does not default within the next T years is given by (see Black and Cox (1976) for a derivation of this formula):

$$P_0(y,T) = N\left(\frac{\ln(y/\underline{y}) + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}\right) - \left(\frac{y}{\underline{y}}\right)^{-2(\hat{\mu}-\sigma^2/2)/\sigma^2} N\left(\frac{-\ln(y/\underline{y}) + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}\right), \tag{20}$$

where the subscript ‘0’ indicates, that recapitalization is not allowed. Therefore, the theoretical expected default frequency in the static case is

$$\text{TEDF}(y,T) = 1 - P_0(y_t). \tag{21}$$

[12] If one is interested in the TEDFs computed with respect to the objective probability measure (objective TEDFs), all formulas in this section can be used after substituting μ (the drift of the underlying cash flow process under the objective measure) for $\hat{\mu}$. However, when choosing the reorganization thresholds, equityholders will find their optimal decision applying the risk neutral valuation presented in previous sections. This means, the critical thresholds $\underline{y}$, $\tilde{y}$, and $\bar{y}$ are determined with respect to the risk-adjusted drift $\hat{\mu}$ (and need, thus, not be adapted when moving from risk-neutral to objective TEDFs).

For a firm that dynamically adjusts its capital structure using the optimal recapitalization strategy $(\underline{y}, \tilde{y}, \bar{y})$, we proceed in several steps. Assuming that the initial inverse leverage ratio is y, we first calculate the probability that the firm neither defaults nor recapitalizes within the next T years. This probability of surviving with a constant debt level is given by (see Appendix A):

$$P_s(y,T) = \sum_{k=-\infty}^{\infty} \Bigg[\left(\frac{\bar{y}}{\underline{y}}\right)^{-2(\hat{\mu}-\sigma^2/2)k/\sigma^2} \big[N(d_1) - N(d_2)\big] - \left(\frac{\bar{y}}{\underline{y}}\right)^{-2(\hat{\mu}-\sigma^2/2)k/\sigma^2} \left(\frac{y}{\underline{y}}\right)^{-2(\hat{\mu}-\sigma^2/2)/\sigma^2} \times \big[N(d_3) - N(d_4)\big] \Bigg], \tag{22}$$

where d_1, d_2, d_3, and d_4 are determined by

$$d_1 = \frac{\ln(y/\underline{y}) - 2\ln(\bar{y}/\underline{y})k + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \frac{\ln(\bar{y}/\underline{y})}{\sigma\sqrt{T}},$$

$$d_3 = \frac{-\ln(y/\underline{y}) - 2\ln(\bar{y}/\underline{y})k + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}, \qquad d_4 = d_3 - \frac{\ln(\bar{y}/\underline{y})}{\sigma\sqrt{T}},$$

and $\mathrm{N}(\cdot)$ denotes the cumulative distribution function of the standard normal distribution.

In the second step we determine the recapitalization density $f(y,t)$, i.e., $1/\mathrm{d}t$ times the probability that a firm with inverse capital ratio y will recapitalize within the time interval $[t, t + \mathrm{d}t]$. This implies that the firm does not default nor recapitalize in the entire interval $[0, t]$ (for the derivation of the equation see Appendix B):

$$f(y,t) = \left(\frac{\bar{y}}{y}\right)^{(\hat{\mu}-\sigma^2/2)/\sigma^2} \mathrm{e}^{-(\hat{\mu}-\sigma^2/2)^2 t/(2\sigma^2)} \times \sum_{k=0}^{\infty} \frac{\ln(\bar{y}/y) + 2\ln(\bar{y}/\underline{y})k}{\sqrt{2\pi}\sigma t^{2/3}} \mathrm{e}^{-(\ln(\bar{y}/y)+2\ln(\bar{y}/\underline{y})k)^2/(2\sigma^2 t)}. \tag{23}$$

In a last step we determine the probability that a firm which is allowed to recapitalize n times will not default within T years using the iteration rule

$$P_n(y,T) = P_s(y,T) + \int_0^T P_{n-1}(\tilde{y}, T-t) f(y,t)\,\mathrm{d}t. \tag{24}$$

Two mutually-exclusive events contribute to this probability. Either the firm survives without recapitalization represented by the first term. Or it recapitalizes at some time t and survives another $T - t$ years starting from $\tilde{y}$. The second term of Eq. (24) integrates over all possible recapitalization times. However, this integration has to be performed numerically. The probability that a firm with dynamic capital structure strategy defaults within the next T years is therefore

$$\mathrm{TEDF}(y,T) = \lim_{n\to\infty} \big[\mathrm{TEDF}_n(y,T) = 1 - P_n(y,T)\big]. \tag{25}$$

Equation (25) allows us to examine the contribution of increasing the number of possible recapitalizations to TEDFs. Our numerical studies show, that this contribution converges very fast to zero. However, the first recapitalization options contribute significantly to the default probabilities.

In the following we compute TEDFs over a time horizon of three years, i.e., $T = 3$. Figure 2 compares the three year TEDF of a firm (we take the base case parameters from Table 2) with a dynamic capital structure strategy to that of a firm with a static debt level. These default frequencies are plotted against the 'distance to default' (DD) which we define as

$$\mathrm{DD}(y) = \frac{\ln(y/\underline{y}) + (\hat{\mu} - \sigma^2/2)T}{\sigma\sqrt{T}}. \tag{26}$$

It measures the distance between the expected firm value at time T and the default trigger, expressed in multiples of the standard deviation (this definition is motivated by the DD measures used in commercial credit risk systems, see KMV, 2001). For the case with static debt level, TEDF is monotone decreasing in the distance to default. Hence, the larger DD, the lower is the default probability (see Fig. 2).

When the firm dynamically adjusts its capital structure, the monotonicity of TEDF is lost. Firms that are performing well have a high probability that they will recapitalize in the near future. When recapitalizing, the inverse leverage ratio jumps from $\bar{y}^*$ to $\tilde{y}^*$ (see Fig. 1), with the consequence that for firms with a dynamic capital structure strategy we have TEDF($\bar{y}^*, \cdot$) = TEDF($\tilde{y}^*, \cdot$). That is, the correspondence from distance to default to TEDF is U-shaped.

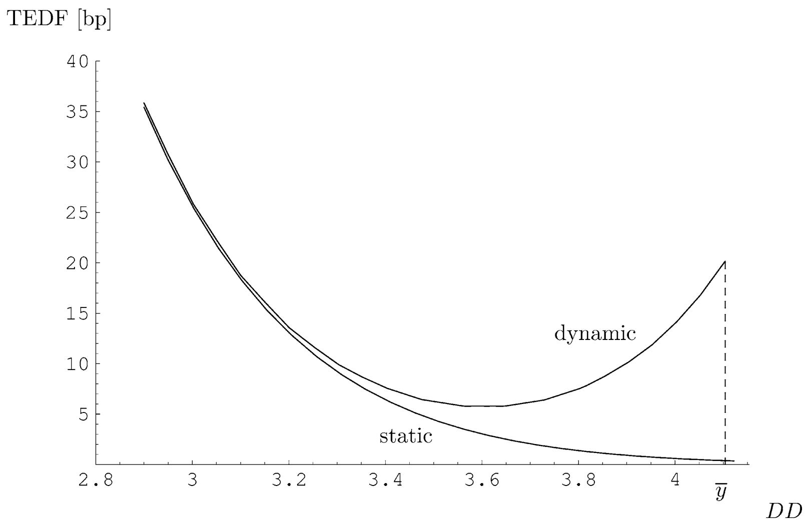


Fig. 2. Expected default frequencies with dynamic and static debt level plotted against the distance to default. While a static debt level leads to a monotonically decreasing relation between DD and TEDF, dynamic capital structure leads to a U-shaped relationship. Debt reorganization at $\bar{y}$ leads to a reduction in the distance to default and therefore increases the default probability when y approaches $\bar{y}$.

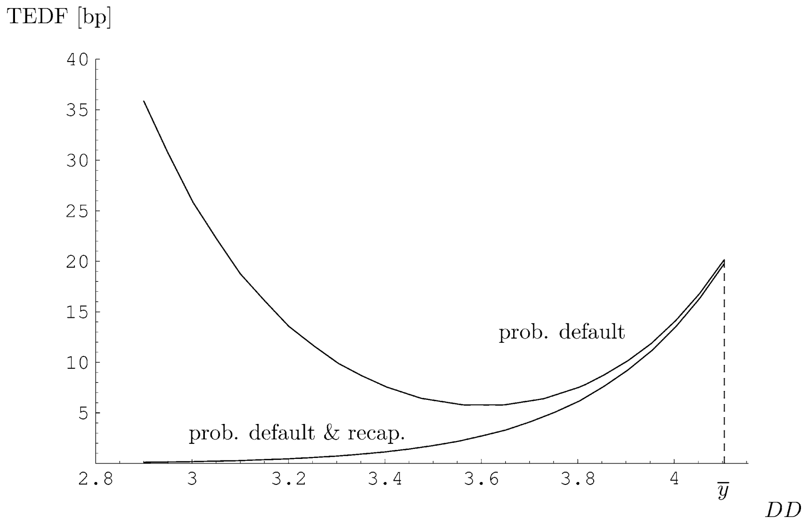


Fig. 3. Expected default frequencies with dynamic capital structure and the unconditional probability that default occurs after recapitalization, i.e., the probability that the firm recapitalizes and defaults subsequent to the reorganization. When y is close to the recapitalization threshold $\overline{y}$, nearly the entire default probability comes from defaults that occur subsequent to a recapitalization.

Figure 3 plots TEDF of the dynamic capital structure strategy together with the probability of paths that first hit the recapitalization trigger and default afterwards. It confirms that for firms with high DD's, nearly the entire default probability comes from these recapitalization paths.

The last two figures (Figs. 4 and 5) shed light on the problem of underestimating TEDF when ignoring the firm's opportunity to adjust its capital structure. Figure 4 shows that the underestimation is more severe for high-risk firms than for low risk firms. Figure 5 illustrates that this effect is more pronounced for high-growth firms.

3.4. Value-at-risk of risky debt

This section focuses on the value-at-risk (VaR) of an investment in a corporate bond. We assume that the bondholders reinvest in the firm's debt after reorganization. That is, after debt is called as well as after bankruptcy, the entire payoff received by the bondholders is reinvested in bonds of the reorganized firm. This is feasible since the firm continues operation after reorganization as an optimally-levered firm (see Section 2).

When calculating the VaR of a debt contract, the dynamics of the underlying y_t (see Eq. (3)) must be translated into the dynamics of $D(y_t)$ in order to determine the respective quantile of the loss distribution. Furthermore, it has to be taken into account that the outstanding principal (and thus the coupon flow) may change after bankruptcy or after a call of the existing bond (when call premium is positive, $\lambda > 0$).

It can be seen from Fig. 6(a) that dynamic capital structure strategies significantly alter the shape of the value function $D(y)$. A dynamic capital structure policy leads to a relatively flat value function and thus, the value of the bond is less sensitive to changes

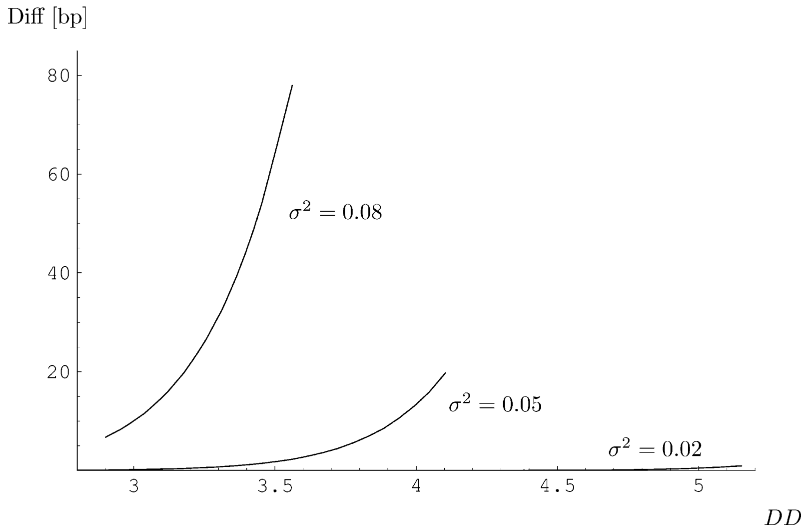


Fig. 4. The underestimation of TEDFs when ignoring the opportunity to recapitalize plotted against the distance to default for different risk levels. The underestimation is higher for high-risk firms.

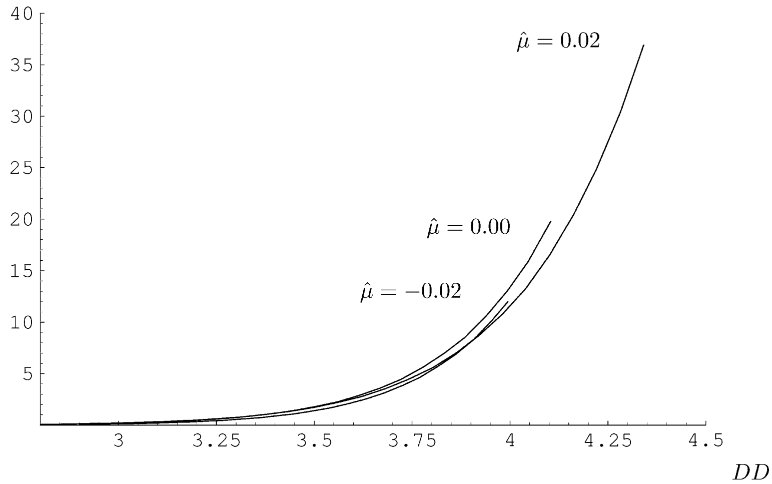
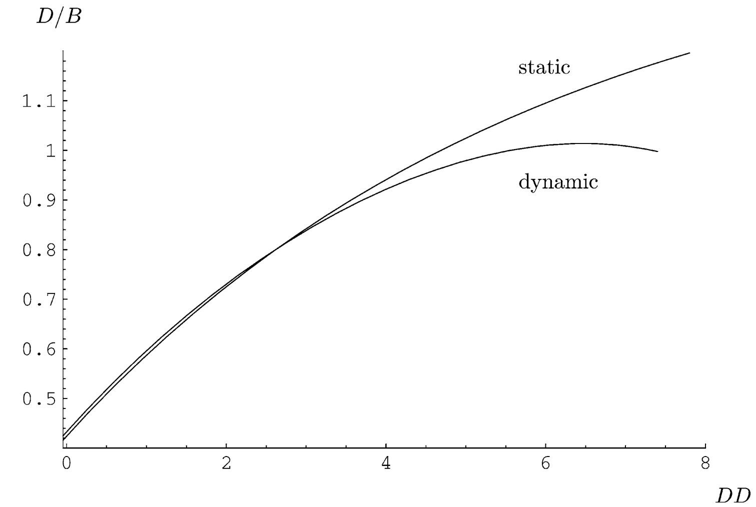


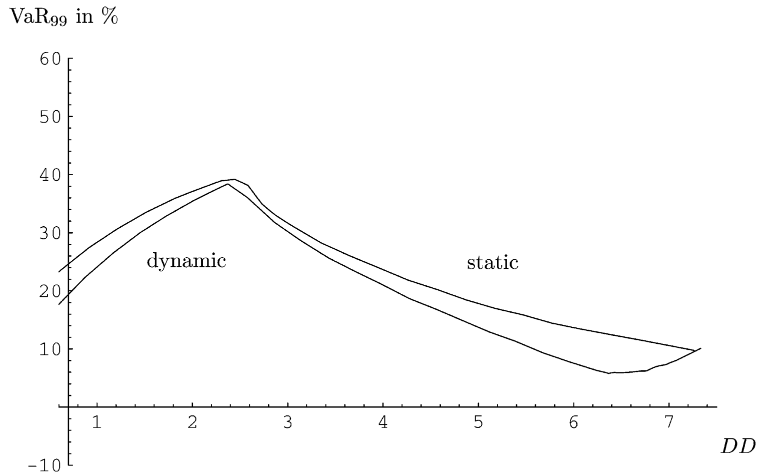
Fig. 5. The underestimation of TEDFs when ignoring the opportunity to recapitalize plotted against the distance to default for different growth rates. The effect is more pronounced for high-growth firms.

in y than under a static capital structure policy. This has implications for the VaR of an investment in the firm's debt securities.

Figure 6(b) plots the typical shape of the one year 99% VaR of an investment into corporate debt for both dynamic and static debt levels versus the distance to default. The functional form of the VaR is somewhat counterintuitive. For very low distances to default the VaR increases with an increase in the distance to default. Thus, bankruptcy defines something like a 'floor value' for the value of the bond. For very low distances to default,



(a)



(b)

Fig. 6. (a) The value of the debt contract for the base case parameters as function of the distance to default for static and dynamic debt level. (b) The one-year value-at-risk at a 99% confidence level of a debt contract for the base case parameters as function of the distance to default for static and dynamic debt level. It is assumed that the investor reinvests in the firm's debt after reorganization, i.e., after debt is called as well as after bankruptcy the entire payoff is re-invested into bonds of the newly (and optimally) levered firm.

the market value of the bond is already close to the bond's value in bankruptcy. However, immediately after bankruptcy the firm will recapitalize and thus the future risk of the bond will actually be reduced. This can also be explained by Fig. 6(a) which is flatter for high distances to default.

For intermediate values of distances to default, the bond trades at a price already significantly above what bondholders would get in bankruptcy. Thus, the "downside potential" of the bond is now higher. Also the probability of a decrease in leverage is now lower. Thus, in this region the VaR increases with the distance to default.

By contrast, for higher distances to default the VaR decreases with the distance to default. The intuition can again be easily seen from Fig. 6(a). The value function for debt becomes flatter for high distances to default, both for dynamic and for static capital structure policies. However, this effect is much more pronounced for the dynamic capital structure policy. As a consequence, the VaR in Fig. 6(b) drops faster for firms with dynamic recapitalization policies than for the case of a static capital structure choice.

Finally, for very high distances to default, the VaR starts to rise again for the case of a dynamic recapitalization policy. This is simply due to the fact that for high distances to default there is an increasing probability of recapitalization. After a leverage-increasing recapitalization, the bond becomes riskier, i.e. the bond value is moved back to the steeper region in Fig. 6(a).

4. Conclusions

This paper has explored the effects of capital structure dynamics on credit risk characteristics of corporate debt. We found that the option to adjust capital structure over time makes firms choose a lower initial leverage ratio. Despite this fact, bondholders generally require higher credit spreads to compensate them for the risk of future leverage increases. This is due to the fact that it is in the equityholders' interest to increase leverage when firm value increases whereas they are reluctant to reduce leverage when firm value decreases.

The numerical analysis also produced estimates for the significance of model risk, i.e. the mistake that is made by using a static model to infer the risk-adjusted drift of the cash flow process from observed equity prices. We found that model risk exacerbates the underestimation of fair credit spreads that results from ignoring capital structure dynamics. The magnitude of the mistake increases with the volatility of the underlying asset value and with the tax benefit of debt and decreases with the premium that must be paid to old bondholders before a leverage increase.

We also analyzed the relationship between the distance to default and expected default frequencies. We found that this relationship is nonmonotonic. While the expected default frequency initially decreases with the distance to default, it actually increases for high values of the distance to default. This happens since the probability of a leverage increasing capital structure adjustment increases with the firm's distance to default. As a result the relationship between expected default frequency and distance to default is U-shaped. Compared to the results from the dynamic model, the static model significantly underestimates credit risk for large distance to default values. This result is consistent with the observation that empirical default frequencies converge much slower to zero than implied by credit risk models which assume static debt levels.[13]

An important implication of our numerical results is that the relationship between a firm's distance to default and its expected default frequency crucially depends on firm characteristics. In particular, the relationship depends on the volatility of the underlying

[13] See, for example, the graph in Crouhy et al. (2000).

cash flow process, the expected growth of the firm's cash flow, and the costs of recapitalization, including the call premium. Thus, the analysis strongly suggests that one should condition on these characteristics when estimating the empirical relationship between the distance to default and expected default frequencies.

Finally, we found that there exists a counterintuitive interplay between capital structure policy and the value-at-risk of corporate bonds. The model predicts that the VaR of corporate bond positions is highest for moderate or intermediate distances to default. It is lower both for extremely low and very high distances to default. Thus, bankruptcy provides an implicit floor value for the bond whereas bonds experience more downside potential for higher distances to default. For very high distances to default, the bond's VaR rises again, due to the high probability of a recapitalization.

Several of our results could be tested empirically. First, expected default probabilities are predicted to be non-monotonic in the firms' distances to default. Second, the non-monotonicity should be particularly pronounced for high-risk firms and high-growth firms. Also, for firms with low effective corporate tax rates, such as firms with large non-debt tax shields, the non-monotonicity should be found to be less significant. Third, the VaR of corporate bonds issued by firms with different distances to default can be calculated from observed corporate bond data. The results can then be compared with Fig. 6(b).

Our analysis emphasizes that bank regulation must take into account dynamic strategies of lenders and financial institutions. Static risk measures especially underestimate banks' risk exposures resulting from loans to apparently highly solvent lenders, since they do not account for the possibility of leverage increasing adjustments. As discussed above, the variables which are most significantly related to the effects of leverage dynamics are the lenders' growth rates, the volatilities of their cash flows and their tax situation.

These results are particularly relevant when empirically calibrated models can be used under Basel II to calculate banks' equity requirements. A lender's current distance to default should not be taken as a sufficient statistic by bank supervisors for the expected default frequency. The latter critically depends on leverage dynamics which in turn depend on the firm characteristics mentioned above. More generally we believe that accounting for firms' and banks' intertemporal responses to a stochastically evolving world is one of the central challenges for efficient bank regulation.

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Appendix A. Derivation of Eq. (22)

Take the geometric Brownian motion (3) and consider the transformation

$$x_t = \frac{1}{\sigma}\ln\left(\frac{y_t}{y_0}\right). \tag{A.1}$$

As long as y_t is between $\underline{y}$ and $\bar{y}$, x_t is a Brownian motion with drift which follows the SDE $\mathrm{d}x_t = \nu\,\mathrm{d}t + \mathrm{d}W$, starting at $x_0 = 0$. The drift is given by

$$\nu = \frac{\hat{\mu} - \sigma^2/2}{\sigma}. \tag{A.2}$$

The lower bound $\underline{y}$ and the upper bound $\bar{y}$ translate to

$$a = \frac{-\ln(y_0/\underline{y})}{\sigma}, \qquad b = \frac{\ln(\bar{y}/y_0)}{\sigma}. \tag{A.3}$$

The probability that x_t does neither hit the lower boundary a nor the upper boundary b within the time interval $[0, T]$ is given by (see Borodin and Salminen, 1996, Eq. (1.15.4), p. 211)

$$P_{x_0} = \frac{\mathrm{e}^{-\nu^2 T/2}}{\sqrt{2\pi T}} \sum_{k=-\infty}^{\infty} \int_a^b \mathrm{e}^{\nu(z-x_0)}\big(\mathrm{e}^{-(z-x_0+2k(b-a))^2/(2T)} - \mathrm{e}^{-(z+x_0-2a+2k(b-a))^2/(2T)}\big)\,\mathrm{d}z. \tag{A.4}$$

Integration leads to

$$P_{x_0} = \frac{\mathrm{e}^{-\nu^2 T/2}}{\sqrt{2\pi T}} \sum_{k=-\infty}^{\infty} 2\left(\mathrm{e}^{2(a-b)k\nu+\nu^2 T/2}\sqrt{\frac{\pi T}{2}}\big[N(d_1) - N(d_2)\big] - \mathrm{e}^{2(a-x_0)\nu+2(a-b)k\nu+\nu^2 T/2}\sqrt{\frac{\pi T}{2}}\big[N(d_3) - N(d_4)\big]\right) \tag{A.5}$$

with

$$d_1 = \frac{x_0 - a + 2(a-b)k + \nu T}{\sqrt{T}}, \qquad d_2 = \frac{x_0 - b + 2(a-b)k + \nu T}{\sqrt{T}},$$
$$d_3 = \frac{a - x_0 + 2(a-b)k + \nu T}{\sqrt{T}}, \qquad d_4 = \frac{2a - b - x_0 + 2(a-b)k + \nu T}{\sqrt{T}}.$$

Substitution of ν, a and b from Eqs. (A.2) and (A.3) into Eq. (A.5) gives Eq. (22).

Appendix B. Derivation of Eq. (23)

Consider the transformation $y_t \to x_t$ defined in Eq. (A.1) above. If the firm starts after n recapitalizations at an inverse leverage ratio y_0 then the recapitalization density is the probability density of the event that it recapitalizes for the $(n+1)$-th time. I.e., it is the probability that y_t hits the recapitalization boundary $\bar{y}$ in the time interval $[t, t + \mathrm{d}t]$ without previously hitting the bankruptcy boundary $\underline{y}$. Translated to the process x it is the probability density of hitting b without hitting a before. This is given by (see Borodin and Salminen (1996), Eq. (3.0.6b), p. 233)

$$f_{x_0} = \mathrm{e}^{\nu(b-x)-\nu^2 t/2} ss_{x-a,b-a}(t), \tag{B.1}$$

where ss is defined by

$$ss_{u,w}(t) = \sum_{k=-\infty}^{\infty} \frac{w - u + 2kw}{\sqrt{2\pi} t^{(3/2)}} \mathrm{e}^{-(w-u+2kw)^2/2t}. \tag{B.2}$$

Substitution of ν, a and b from Eqs. (A.2) and (A.3) gives Eq. (23).

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